Lecture II: Liquidity, Price Impact, and Resiliency

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References:

P. Weber and B. Rosenow, Orderbook approach to price impact, eprint cond-mat/0311457, Quant. Fin. 05

P. Weber and B. Rosenow, Large Stock price changes: volume or liquidity?, eprint cond-mat/0401132, Quant. Fin. 05
Why is it interesting - stock prices as a random walk?

large fluctuations more frequent than expected for a gaussian distribution


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Why is it interesting: power law distribution for stock returns

\[ G(t) = \ln S(t + \Delta t) - \ln S(t), \quad \Delta t = 1 \text{min} \]

Cumulative probability distribution \[ P(g > x) \sim 1/x^3 \]

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Outline

• Why and how stock prices change
• Large returns and liquidity fluctuations
• Microscopic structure of large returns
Liquidity

• Concept of market liquidity describes, how “easy” a financial instrument can be bought or sold, encompasses various transactional properties of markets.

• Market depth denotes the amount of order flow innovation which is required to change prices a given amount.

• Resiliency describes the speed with which prices recover from a random uninformative shock, and

• Tightness is the cost for turning around a certain amount of shares within a short period of time.
Reasons for price changes I

• Present value of a company is discounted sum of future dividends/earnings

• Information about economic situation of a company influences expectations about future earnings $\rightarrow$ value of the company

• News/Information influences stock price
Reasons for Price Changes II

• Supply and demand influence price

• Measurement of supply and demand by difference $Q$ between the number of stocks bought and the number of stocks sold („volume imbalance“)

• Price impact function describes the relation between return $G$ and volume imbalance $Q$

• Efficient market: only the information content of $Q$ influences price; possibly incorrect
Standard price formation equation

\[ S(t + \Delta t) - S(t) = \sum_{t_i \in [t, t+\Delta t]} \lambda_{t_i} Q_{t_i} + \sum_{t_i \in [t, t+\Delta t]} u_{t_i} \]

- \( S(t) \): stock price at time \( t \)
- \( q_{t_i} \): volume (number of stocks) of transaction \( i \)
- \( \lambda_t \): instantaneous price impact (inverse liquidity)
- \( u_t \): noise term describing the arrival of new information
Literature related to price impact

Hasbrouck (1991)  concave price impact
Hausmann, Lo, MacKinley (1992)  ordered probit model
Kempf und Korn (1999)  description of nonlinear effects
Zhang (1999)  square root price impact
Dufour und Engle (200)  time and price impact
Rosenow (2002)  liquidity and volatility
Evans und Lyons (2002)  Q determines exchange rates
Hopman (2002)  mechanical price pressure
Lillo, Farmer, Mantegna (2003)  master curve for price impact
Gabaix, Gopikrishnan, Plerou, Stanley (2003)  large G from large Q (*)
Potters und Bouchaud (2003)  permanent price impact
Bouchaud, Gefen, Potters, Wyart (2003)  correlated Q, uncorrelated G
Lillo und Farmer (2003)  criticism of (*)

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**Data Sets**

- Island ECN order book for 2002, 20% of NASDAQ volume
- 10 most frequently traded stocks like Cisco, Microsoft, Oracle (AMAT, BRCD, BRCM, CSCO, INTC, KLAC, MSFT, ORCL, QLGC, SEBL)
- TAQ data base published by the New York stock exchange
- 44 most frequently traded NASDAQ stocks
- all trades and quotes in the year 1997
Order book

• information: all limit orders

• complete description of stock market:
  • limit orders
  • market orders

• bid price = highest buy limit order

• ask price = lowest sell limit order

• market orders ("marketable limit orders") execute limit orders

• exclude transactions including „hidden“ limit orders

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Normalization of returns and volumes

• generate transaction data from order book

• (midquote) return \( G_M(t) = \ln S_M(t + \Delta t) - \ln S_M(t) \)

• volume \( Q(t) = \sum_{i=1}^{N_{\Delta t}} q_i \) \( \Delta t = 5\text{min} \)

• normalize returns by standard deviation \( \sigma_G \) and volumes by \( \sigma_Q = \langle |Q - \langle Q \rangle| \rangle \)

→ different stocks are comparable, analysis of ECN and TAQ data are comparable
Average price impact

\[ I_{\text{average}}(Q) = \langle G \rangle_Q \]

as a conditional expectation value

\[ G = 0.48 \cdot Q^{0.76} \]

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Description of order book

- measure prices logarithmically form bid/ask price
- describe order book by density function $\rho(\gamma, t)$ with

$$
\gamma = \begin{cases} 
\ln(S_{\text{limit}}) - \ln(S_{\text{bid}}) & \text{buy limit order} \\
\ln(S_{\text{limit}}) - \ln(S_{\text{ask}}) & \text{sell limit order}
\end{cases}
$$

- reconstruct density function from information about placement, cancellation and execution of limit orders
Average order book

for details see work by Maslov, Challet and Stinchcomb, Bouchaud group, Farmer group

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Virtual price impact

Integration of order book

\[ Q = \int_0^G \rho(\gamma) \, d\gamma \]

yields \( Q(G) \)

by inverting this relation one obtains

\[ G = I_{\langle \text{book} \rangle}(Q) \]

\[ G = 1.22 \left( Q_{\text{book}} \right)^{1.19} \]
Averaging and inversion

Problem: $\langle G \rangle_Q$ cannot be calculated by inverting $\langle Q \rangle_G$

Hence: invert

$$Q(t) = \int_{0}^{G(t)} \rho(\gamma, t) \, d\gamma$$

and average afterwards

$$\langle I_{\text{book}} \rangle(Q)$$
dominated by rare events with low liquidity

typical price impact better described by $I_{\text{book}}(Q)$
Correlations between return and order flow I

Correlation function

\[ c_\alpha(\tau) = \frac{\langle Q_\alpha(t + \tau)G(t) \rangle - \langle Q_\alpha(t) \rangle \langle G(t) \rangle}{\sigma_{Q_\alpha} \sigma_G} \]

\( \alpha = \text{market market orders} \quad Q_{\text{market}} \quad \text{in interval} \quad \delta t = 50s \)

\( \alpha = \text{limit limit orders with} \)

\[ Q_{\text{limit}}(t) = \int_{-\infty}^{\infty} \text{sign}(-\gamma)(Q_{\delta t}^{\text{add}}(\gamma, t) - Q_{\delta t}^{\text{canc}}(\gamma, t)) \, d\gamma \]

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Correlations between returns and order flow II

market orders correlated with returns

limit orders anticorrelated with returns

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Correlations between returns and order flow III

• anticorrelations between returns and limit orders describe market resiliency: recovery from uniformative shocks

• value traders become active only at the outside spread

• reserve orders are not visible in the order book, order management systems like Archipelago place new orders if the old ones are executed

• compare Bouchaud et al.: transitory price impact
Correlation volume

- average order flow \( \rho_{\text{flow}}(\gamma) = \langle Q_{\Delta t}^{\text{add}}(\gamma) - Q_{\Delta t}^{\text{canc}}(\gamma) \rangle \)

integration of \( \rho_{\text{flow}} \) yields \( Q_{\text{flow}}(G) \)

- order flow due to correlations

\[
\rho_{\text{corr}}(\gamma, G') = \langle Q_{t_0}^{\text{add}}(\gamma) - Q_{t_0}^{\text{canc}}(\gamma) \rangle_{G'} - \langle Q_{t_0}^{\text{add}}(\gamma) - Q_{t_0}^{\text{canc}}(\gamma) \rangle
\]

\( \rho_{\text{corr}}(\gamma, G') \) saturates for \( t_0 \geq 30 \text{ min} \)

integration of \( \rho_{\text{corr}}(\gamma, G') \) yields \( Q_{\text{corr}}(G') = \int_0^G \rho_c(\gamma, G') d\gamma \)
Approximation of „stationary returns“

Assume \( G(t) \equiv G \),

total volume

\[
Q(G) = Q_{\text{book}}(G) + Q_{\text{flow}}(G) + Q_{\text{corr}}(G)
\]

• calculate price impact by inverting \( Q(G) \)

• good agreement between “theory” and empirical data
Analysis of extreme events - TAQ data base

- 44 most frequently traded NASDAQ stocks
- all trades and quotes in the year 1997

trades:

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Classification of Transactions

• $S_A(t)$ ask price
• $S_B(t)$ bid price
• $S_M(t) = \frac{1}{2}[S_B(t) + S_A(t)]$ midquote price
• algorithm of Lee & Ready (1991)

• $S(t) < S_M(t)$ seller induced transaction, $\epsilon_i = -1$
• $S(t) > S_M(t)$ buyer induced transaction, $\epsilon_i = +1$
Typical data errors

• recording errors, e.g. decimal point at wrong position (98.0 → 9.80)

• artefacts due to combination of different ECNs (Electronic Communications Networks)
Large returns - filter algorithm for TAQ data

Discard all transactions with

• transaction price < 0
• bid-ask spread < 0
• bid-ask spread > 40% transaction price
• transaction price – midquote price < 4 · bid-ask spread

Large events and average price impact

TAQ, 1198 events
returns \( G > 5\sigma_G \)
Island order book,
210 events

average price impact has weak explanatory power for large returns

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Liquidity measures I: depth and tightness

- **depth** $D = \int_{0}^{5\sigma_G} \rho(\gamma) \, d\gamma$ is size of market order required to change price by $5 \sigma_G$

- **tightness** $T$ is cost of round trip (buying and selling volume $2 \sigma_Q$ over short period of time)

  $$T = \frac{1}{|I_{\text{book}}(2\sigma_Q)| + |I_{\text{book}}(-2\sigma_Q)|}$$

- return predicted from average price impact by

  $$G_{\text{pred}}(t) = \tilde{I}_{\text{market}}(\tilde{Q}(t))$$
Explanation of extreme returns by depth and tightness

R² = 0.14  
R² = 0.11

depth and tightness have explanatory power for large returns

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Liquidity measures II: dynamic liquidity

- $\rho_{\text{book}}(\gamma, t)$ density of limit orders in the book in the beginning of 5 minute interval

- $\rho_{\text{flow}}(\gamma, t)$ density of limit orders added to book within 5 minute interval

\[
\tilde{Q}(G) = \int_0^{5\sigma_G} (\rho_{\text{book}}(\gamma) + \rho_{\text{flow}}(\gamma)) \, d\gamma
\]

calculate $\tilde{I}_{\text{actual}}(\tilde{Q})$ by inverting this relation

- determin slope $\chi(t)$ of $\tilde{I}_{\text{actual}}(\tilde{Q})$ by linear fit in region $0 \leq G \leq 5 \sigma_G$
Actual price impact for some extreme events
Dynamic liquidity and large returns

- large returns mostly due to low liquidity (steep price impact function)

- compare Farmer et al. cond-mat/0312703: large returns on tick basis explained by gaps in the order book
Microscopic structure of large returns

- study intervals with fixed number $N=100$ trades

- tick return $\delta g_i = \ln(s_{i+1}) - \ln(s_i)$

- total return $G = \sum_{i=1}^{N} \delta g_i$

- average tick return $\delta g = \frac{1}{n} \sum_{\delta g_i \neq 0} |\delta g_i|$

- $N_+ (N_-)$ trades with nonzero return in (against) the direction of $G \rightarrow$ with nonzero $\Delta N = N_+ - N_-$
Direction versus average size of tick returns

Direction of tick returns more important than size

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Conclusion

• difference between virtual and average price impact due to resiliency

• large returns mainly due to small liquidity

• intervals with large returns: many price changes in the same direction