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The Investors Game: A Model for Coalition Formation

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Abstract

We propose a multi-agent system in which agents form coalitions to realize common investment projects. Each project is conducted by an initiatior who has to convince other agents to invest until a threshold value is reached. The decision of an agent to invest depends on the previous experience with the particular initiator, i.e., on the failure or success of the projects conducted by him. We investigate the formation of coalitions, i.e., networks of partners that repeatedly invest together and the lifetime of these coalitions. Further, we discuss how the dynamics depends on crucial parameters such as the memory or the risk disposition of the agents.

1 Introduction

Economic multi-agent systems (MAS) recently gained much attention under the label ACE – *agent-based computational economics.* For example, Tesfatsion [1, 2] proposed ACE models to study the relationship between market structure and worker-employer interaction networks. Here workers and employers repeatedly search preferred worksite partners. Their worksite interactions are modeled as prisoner's dilemma games, they evolve their worksite strategies over time based on their earnings in past worksite interactions. Tesfatsion uses descriptive statistics to study correlations between market structure and worker-employer network formations. Two factors investigated are job concentration (number of workers to number of employers) and job capacity (total potential job openings to total potential work offers). If the job capacity is fixed, changes in job concentration produce only small and unsystematic effects on relative market power levels. But the network distribution exhibits two or three sharp isolated peaks corresponding to distinct types of worker-employer interaction networks, that means, the interaction effects are strong. Maybe this strong interaction effects could help to explain the "excess heterogeneity" problem observed in labor markets.

Recent models on the endogenous formation of trade networks are proposed by Albin and Foley [3], Tesfatsion [4] and Vriend [5]. A key concern in these studies is the emergence of a trade network among a collection of buyers and sellers, who adaptively select their trade partners. Agents perform these selections by looking at their past experiences with these partners. Kirman and Vriend [6] show an ACE model of the wholesale fish market in Marseilles. Their interest is to understand the buyer loyalty to sellers by means of repeated business. Price dispersion and loyalty emerge as a result of the co-evolution of buyer and seller decision rules. They find that buyers learn to be loyal as sellers learn to offer a higher payoff to loyal buyers.

In this paper, we present an ACE model of a game where agents want to establish projects and coalitions with initiators of projects. Our focus is on investor-initiator relations during the game. We are interested in questions like: What drives the formation of interaction networks? How do these networks evolve over time? To elucidate these questions, we present computer simulations and also investigate the significance of the different parameters in our model.

2 Rules of the InvestorsGame

We consider a *multi-agent system* of N agents, each possessing a *budget* $e_k(t)$ that can be changed in the course of time:

$$e_k(t+1) = e_k(t) + r_{mk}(t)\, i_k(t) \tag{1}$$

Note that t is measured in discrete time steps, $i_k(t)$ denotes an *investment* of agent k, i.e. an amount of money taken from the budget to spend it on a certain investment project m, and $r_{mk}(t)$ denotes the return on investment (RoI) or the yield from that particular investment, $r = -1$ would mean a complete loss of the investment, which is a lower boundary. Both $r_{mk}(t)$ and i_k may vary in time, with a minimum value $i_{\min}$ equal to all agents. If we further assume that the investment is a particular ratio of the whole budget of the agent, we have:

$$e_k(t+1) = e_k(t)\Big[1 + r_{mk}(t)\, q_k(t)\Big]\ ; \quad i_k(t) = q_k(t)\, e_k(t) \tag{2}$$

Each agent starts with the same inital budget e_0. If its budget e_k in the course of time falls below the minimum investment value $i_{\min}$, the agent is considered "bankrupt", i.e. it will no longer participate in the game. This means that the number of agents actively participating decreases in the course of time. In order to keep N constant, a variant of the game would be to replace the bankrupt agent by a new one that starts with the initial budget e_0.

In order to launch a particular investment project m at time t, a certain minimum amount of money $I_{\rm thr}$ needs to be collected among the agents. The existence of the threshold value $I_{\rm thr} \gg i_{\rm min}$ will force the agents to *collaborate* until the following condition is reached:

$$I_m(t) = \sum_{k \in N_m} q_k(t)\, e_k(t) \geq I_{\rm thr} \tag{3}$$

N_m is the number of agents collaborating in the particular investment project. This is not a fixed number because a small number of "wealthy" agents possessing a larger value of i_k can reach the theshold as well as a larger number of small investors. There may be different investment projects m at the same time, but at the moment it is assumed that each agent participates in only one investment project at a time.

Thus, the first essential feature of the game to be noticed is the establishment of a coalition of agents investing in the same project. The decision of agent k to collaborate in the project m will depend on the previous history it has gained with other agents. For example if agent k is asked by agent j (the initiator of investment project m) to collaborate in a common project, it will check its records of previous encounters. Assuming, it has received a positive or negative payoff,

$$p_{kj}(t) = i_k(t) r_{jk}(t) = e_k(t)\, q_k(t)\, r_{jk}(t) \tag{4}$$

from an interaction with agent j at time t in the past then this will be counted for the decision with a certain weight:

$$w_{kj}(t) = \sum_{n=1}^{H} p_{kj}(t-n)\, e^{-\gamma(t-n)} \tag{5}$$

Here, n counts the previous time steps and H is the horizon of the memory. The double index p_{kj} shall ensure that only those contributions received from interaction with agent j shall be counted for the weight $w_{kj}(t)$. The payoff at time $(t-n)$ may have resulted from the collaborative action of different agents, these however are unknown to agent k, it only realizes the initiator of the project, agent j. Thus, at any given time t each agent k counts on the information of the previous interactions stored in an array $\boldsymbol{W}_k(t)$ of size N which is updated in every time step. For computational reasons it will be convenient to use a function with exponential smoothing instead of the sum in eq. (5):

$$w_{kj}(t+1) = p_{kj}(t) + w_{kj}(t)\, e^{-\gamma} \tag{6}$$

In general, the game proceeds as follows: At any time step t an agent j (the initiator) will be randomly (i.e. with a probability $1/N$) chosen from the ensemble of N agents to launch an investment project I_m. An alternative version of the game used in this paper considers a fixed selection of initiators, which are chosen at start time and then remain the same as the game proceeds. The initiator randomly contacts the remaining agents to convince them to invest in the project until it has collected at least the threshold amount I_{thr}. The contacted agents may accept the offer only with a certain probability:

$$\tau_{kj}(t) = \frac{e^{\beta w_{kj}(t)}}{\max_{w_{kj}(t)} e^{\beta w_{kj}(t)}} \tag{7}$$

that in terms of the w_{kj} considers the good or bad previous experience with agent j, max{} denotes the maximum value of all weights of agent k, and β denotes an additional weight constant. For the decision, agent k draws a random number RND in the interval $[0, 1)$ (zero actually has to be excluded) and accepts the offer if $\tau_{kj} > RND$. Note that because of the normalization the offer from the initiator with the highest $w_{kj}(t)$ (i.e. the one with whom agent k had the best experience) will be always accepted.

Initiator j stops to contact agents if either the investment project has reached the threshold or if all agents that are not already bankrupt are part of the project (which then however fails because it has not reached the threshold).

If the investment project could be established, it will be evaluated. Let us define the RoI for the investment project as:

$$r_m(t) = \frac{\Delta I_m}{I_m} \tag{8}$$

where ΔI_m is the gain or loss of the investment project m. A complete loss of the investment would mean $\Delta I_m = -I_m$, i.e. $r_m = -1$. The evaluation should in general involve certain "economic" criteria that also reflect the nature of the project. However, we do not want to include such assumptions already at the current level of the game, therefore, we simply assume that the failure or success of an investment project I_m is randomly drawn from a probability distribution:

$$r_m(t) = \Big(2\text{RND} - 1\Big)^3 \tag{9}$$

that is also shown in Fig. 1. RND is a random number drawn from the interval $(0, 1)$. For $RND < 0.5$, the project receives a negative yield, while for $RND \geq 0.5$, the project gets a gain. Note that in this assumption both the loss and the gain are bound to the maximum value $r = \pm 1$.

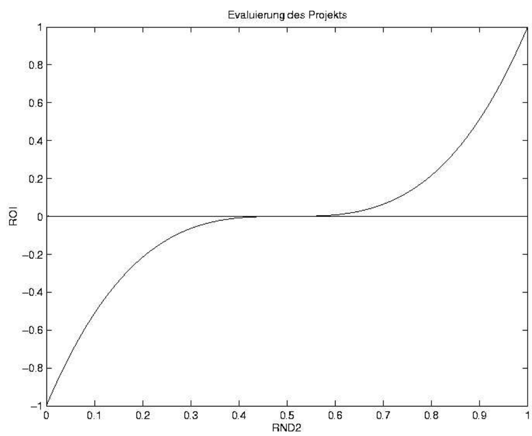

Figure 1: Return on Investment (RoI), eqn. (9)

In order to keep the total amount of invested money constant *in the average*, we have assumed a symmetric distribution of gains and losses here. A more realistic assumption would include also gains with $r \gg 1$, while the loss is still bound to the maximum investment value. In this case, the cummulative distribution of gains and losses should reach about the same value to keep the total amount of money constant *in the average*, otherwise a decrease or increase of the total budget should result.

The gain or loss of the investment project is shared between all agents of the respective coalition proportional to their investment according to eq. (2), where $r_{mk} = r_m$ if agent k has collaborated in the investment project m, i.e.

$$\Delta I_m = I_m \, r_m = \sum_k q_k \, e_k \, r_{mk} \tag{10}$$

All records are then updated accordingly. The game ends if one project cannot be established even though all non-bankrupt agents would have invested on it.

To include a behavioral component in the game, we may further assume that the failure or success of the previous investment has some influence on the risk the agents take in the next step. I.e., big losses (gains) may have a negative (positive) influence of the amount $q_k(t)$ invested during the next round. Such a feedback process between negative/positive experiences and risk aversion/acceptance can be substantiated by psychological arguments and will enhance the dynamics of the game. For simplicity, we choose:

$$q_k(t+1) = b_k q_k(t) \; ; \qquad b_k(t) = a^{r_k(t)} \tag{11}$$

$a = 2$ is chosen. q_k is bound to a maximum value of 1.0, because agents cannot have debts in the current version of the game.

3 Results of Computer Simulations

3.1 Dynamics of Investment

To elucidate the overall dynamics of the InvestorsGame, we first present results obtained from only one computer simulation. As outlined above, the following parameters have to be chosen in addition to the number of agents, N_a, and of initiators, N_i: $I_{\rm thr}$ for the investment value, e_0 and $i_{\rm min}$ for the initial budget and the minimum investment, q_0 for the initial investment ratio, γ for the memory, β for the weight of the acceptance probabilities. For the results presented in the following, we have used: $N_a = 30$, $N_{ini} = 4$, $I_{thr} = 15$, $e_k(0) = 10$, $i_{min} = 0.1$, $q_k(0) = 0.1$, $\gamma = 0.01$, $\beta = 0.1$.

The numerical results for the sample simulation are summarized as follows: The game lasted 1519 time steps. Only in 1071 time steps a project could be established, while it failed in 448 time steps. This is because agents modified their preferences and didn't invest enought money, or they didn't want to invest due to a negative weight value. At the end only 5 agents have survived, that's why there is also a decay of the total budget from 300 to 54.5. The mean time agents remained in coalitions was about 115 time steps (minimum/maximum: 1/183).

Fig. 2 (top) shows how the investment $I_m(t)$ for each established project, eqn. (3), the number of investors (middle) in the course of time and (bottom) the return on investment, eqn. (8), changed in the course of time. Note that the RoI was drawn from the probability distribution, eqn. (9). There is no *direct* correlation between the RoI and the investment, but an *indirect* one because a repeated success of projects lead to a decrease of risk aversion and thus to an increase of the amount invested by successful agents. Therefore, after a sequence of positive RoI, we observe a sudden increase of I_m which then becomes much higher than the threshold needed to establish the project. Note that as the time advance the number of investors decreases i.e. the game is played by only a few very wealthy agents. These agents are also very risky loosing a lot of money during some repeated failure projects. Therefore, after $t = 800$, the number of investors is reduced to less than 7 because most of the agents are already bankrupt.

Fig. 3 (top) shows the change of the total budget, $E(t) = \sum_k e_k(t)$. We observe that in a certain time lapse, between 600 and 800, the total budget increases until a sudden serious decrease in time step $t = 820$, due to a very unsuccesful project. To give an example of the individual agent dynamics, Fig. 3 (middle) shows the budget changes for the most succesful investor, agent number '10', and (bottom) the most unsuccesful investor, agent '16'. Interestingly, the successful agent started to considerably invest only after $t = 400$, whereas at the same time the unsuccesful agent already faced a considerable decay in its budget. Thus, we are tempted to deduce that a good strategy would suggest not to invest a lot at the beginning of the game, but to wait some time steps like agent No. '10' did. But, such conclusions do not pay attention to the randomness of the game – which next time could make just the opposite true.

3.2 Structure of coalitions

An important question of interest in the InvestorsGame is about the formation of networks of agents investing together, which we call *coalitions* here. The ultimate goal of a coalition is to establish a common project. Thus, in the current version of the game, each coalition is centered around one of the project initiators chosen initially. In order to define a coalition, it is not sufficient just to look at the current project, because also agents currently not taking part in a project may have a long-lasting (positive) relationship to a particular investor. Therefore, we rather use the weights $w_{kj}(t)$, eqn. (6), that describe the previous history of agent k with investor j. By definition, agent k is part of a coalition with investor j, if the value w_{kj} is (i) positive and (ii) greater than a threshold value ε. The latter one is needed because of the exponential decay of the memory, that would ensure a small positive (or negative) value of w_{kj} for $t \to \infty$ even in cases where agent k does nothing any more (because it is bankrupt, for instance).

Fig. 4 shows how long two different sample agents, '0' and '13', remain in coalitions established by the four initiators displayed on the Y axis. A positive "pulse" means that the agent is part of one of the possible coalitions, i.e. $w_{kj} > \varepsilon$. We notice that because of positive experiences with different investors, an agent can be part of different coalitions

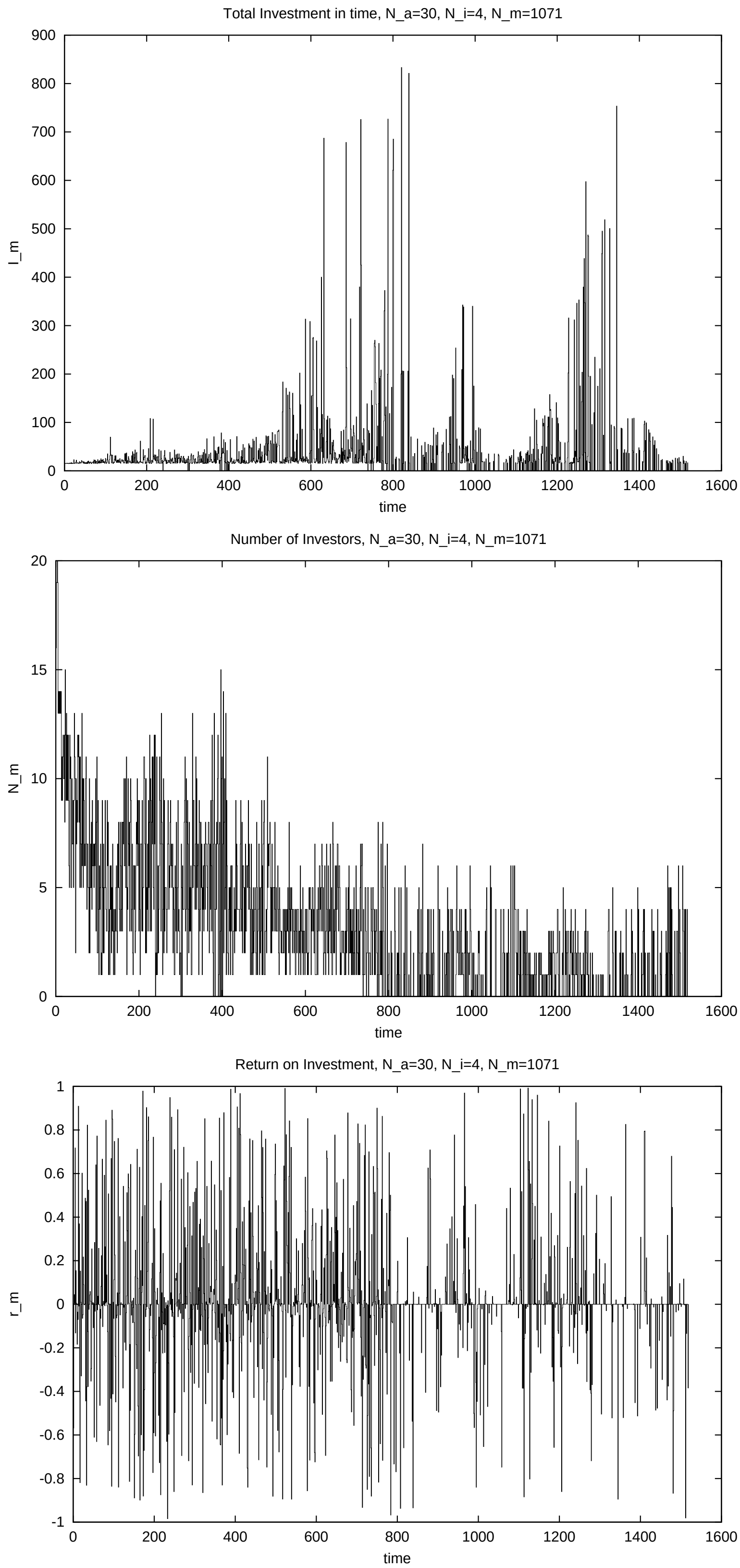

Figure 2: Investment (top), number of investors (middle), RoI (bottom) vs time

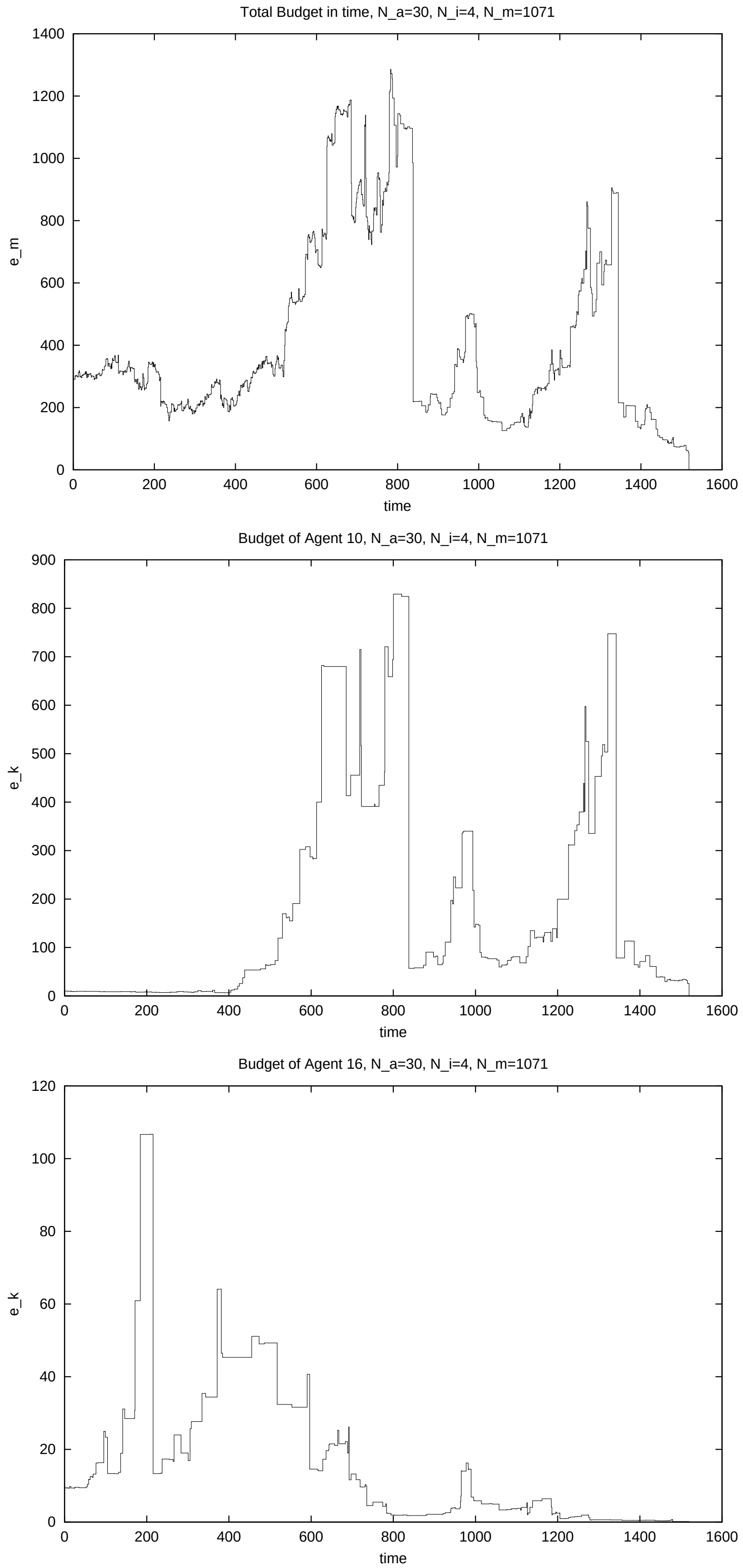

Figure 3: Total budget (top), most successful (middle), unsuccessful (bottom) investor vs time

at the same time, even if it invests only in one project at a time. On the other hand, how long an agent remains in a coalition not only depends on successful projects, but also on the memory, γ, which will be discussed in the next section.

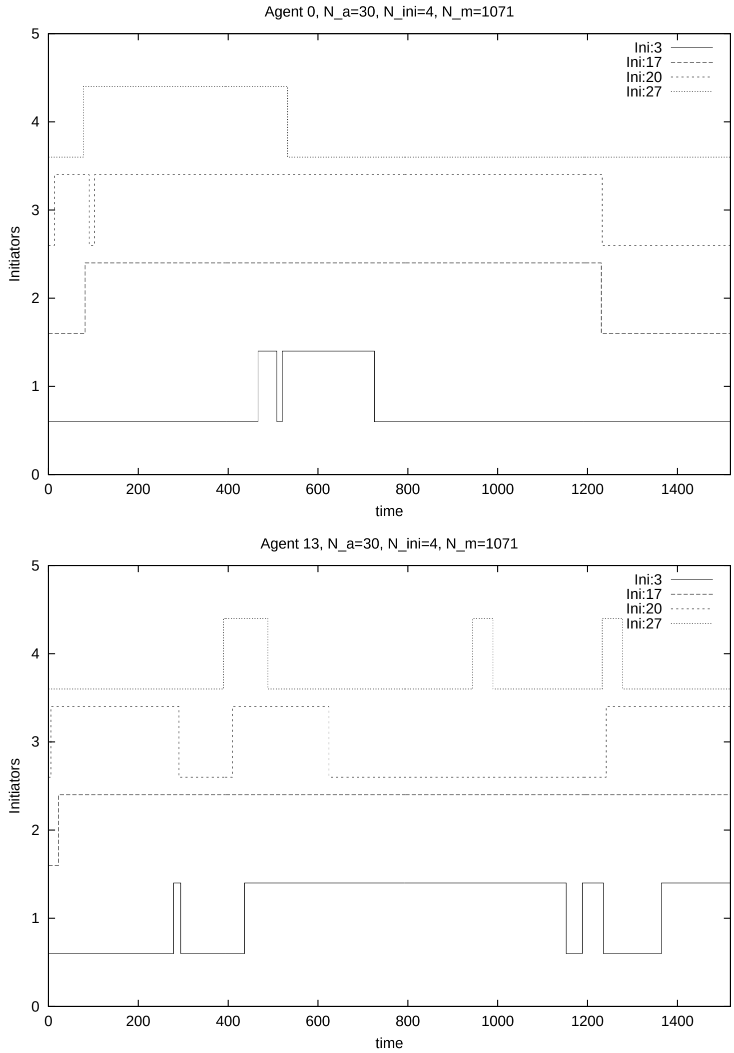

Figure 4: Time agents '0' (top) and '13' (bottom) spent in the coalitions of inititiators '3', '17', '20', '27'. Parameters: $\varepsilon = 0.1$, $\gamma = 0.01$

In order to elucidate the dynamics of the coalitions, we have chosen a graphical representation with polar coordinates. The center represents a particular initiator. The relation of each agent k to this initiator is expressed by the two variables $\varphi = k\,360/N$ and $r = \exp\{-w_{kj}\}$. I.e., φ is used to identify the different agents, while r gives a measure how "close" the relation between agent k and initiator j is. If there is *no* relation, i.e. if $w_{kj} = 0$, then $r = 1$. That means, the circle of $r = 1$ shown in the graphs of Fig. 5 and Fig. 6 simply distinguishes between all those agents who are part of the coalition established by a particular initiator (*insiders*) and those *not* part of that coalition (*outsiders*). Note that agents with a particular negative experience with a certain investor because of failure projects are far away from the circle, while those with *no* significant relationship, i.e. $|w_{kj}| < \varepsilon$, are *close* to the circle of $r = 1$ (and are therefore omitted in Figs. 6 and 5).

Fig. 5 shows two snapshots of how the coalitions for initiator '3' change in time. One can notice, for example, that agents '1' and '22' left the coalition during that time period. Fig. 6 shows snapshots of the coalitions established by the four initiators at the same time step, $t = 1000$. One can see for example that initiator '17' has formed a very strong coalition, because many agents are found very close to him. Initiators '20' and '27', on the other hand, have already some "enemies", i.e. agents who had a very bad experience with them and thus will presumably invest with them never again.

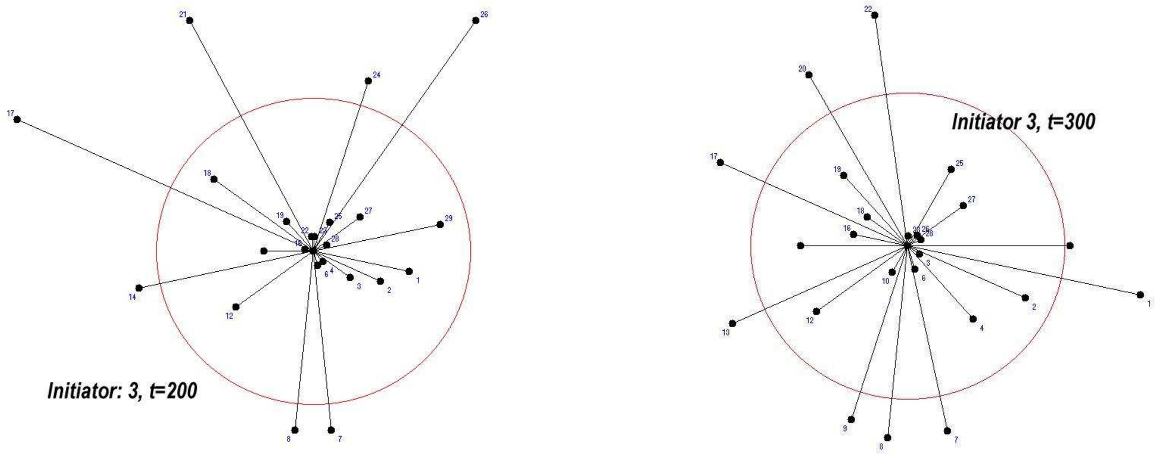

Figure 5: Coalition established by initiator '3' shown at two different time steps: (left) $t = 200$, (right) $t = 300$. Only agents with $|w_{k3}| > \varepsilon$ are plotted.

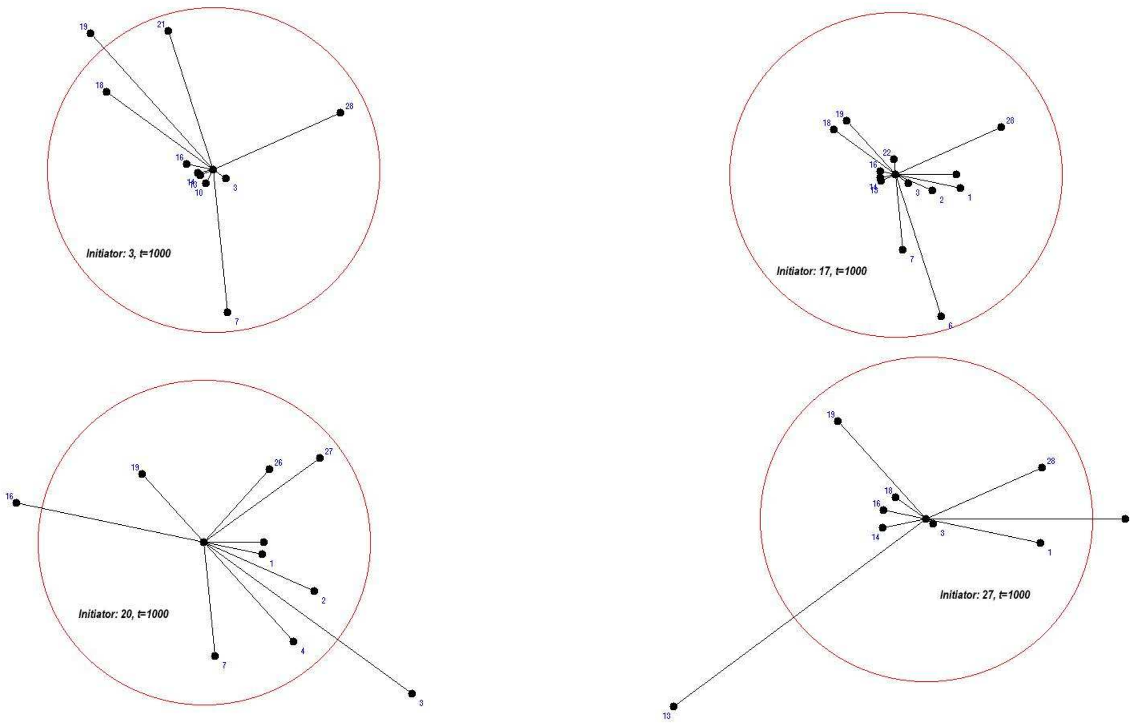

Figure 6: Coalition established by the four different initiators shown at time $t = 1.000$. Only agents with $|w_{kj}| > \varepsilon$ are plotted.

4 Conclusions

In this paper we have shown how feedback processes based on positive or negative experience may lead to the establishment of networks among agents. For simplicity, we have just assumed a random selection of failure or success, but we note that more elaborated economic assumptions, such as market dynamics based on supply and demand, can be taken into account as well. The feedback between the previous experience of an agent and its further investment behavior occurs in a twofold manner:

1. The decision of agent k to join an investment project of initiator j is mediated by a weight function, w_{kj}, eqn. (6), that includes both the most recent gain or loss of a common project and a memory of the previous history. The agent then compares this weight with its best experience at the given time, the ratio of which determines the probability to accept the project.

2. A positive/negative experience further affects the agents' risk acceptance/aversion, q_k, eq. (11). I.e., agents with a repeated success in investment projects have a tendency to invest larger portions of their budget, while repeated failures will cause the agents to reduce the portion of their investment.

In this paper, we have mainly concentrated on the first feedback, describing the establishment and reinforcement of relations among agents and initiators. This is considered a "social component" of the agents' interaction, while the second feedback would rather describe an "behavioral component" of the agent itself.

We were interested in the structures of the networks that appeared between agents and project initiators. We could show that the interaction described above leads to the formation of coalitions, i.e. the stable linkage of a certain number of agents to a particular initiator. Whether or not an agent belongs to a coalition was determined by the weight function, which has to be positive and larger than a certain threshold: $w_{kj} > \varepsilon$. I.e. a positive experience with a given project initiator causes the agent to further "trust" him and to continue to invest in his projects. However, as we have also shown, each coalition only has a certain finite lifetime, because the random occurence of failure projects eventually causes the agents to leave the coalition.

As already outlined above, the memory of an agent plays an important role in determining the structure and the lifetime of a coalition. In our model, the memory is represented by the parameter γ that in eqn. (6) describes the exponential decay of the past experience. Without memory, i.e. $\gamma \to \infty$, agents just randomly gather for a certain project, so we do not observe the formation of networks. Moreover, since they easily forget about their good or bad experience with the different initiators, agents simply invest most of the time until their budget is gone. So, this limit describes the *random* scenario. On the other hand, if agents' memory is too long, i.e. $\gamma \to 0$, then any positive and negative experience will last forever and changes in the structure of the networks are hardly observed. So, this limit describes the *frozen* scenario.

The influence of the second feedback process via risk aversion, mentioned above, will be investigated in more detail in a forthcoming paper. Here, we only mention that this feedback is important to explain the outbreak of larger fluctuations and the sudden end of the game. After a "quiet" initial period characterized by a rather slow increase and small fluctuations both of the investment I_m and the total budget $E(t)$, we observe a period of larger activity characterized by large fluctuations in I_m, Fig. 2, and $E(t)$, Fig. 3. This basically results from the fact that agents, after having experienced some successful projects, start to increase their investment ratio because their fear to loose is less. This in turn increases their losses if the project fails by accident. So, the investment and total budget both decrease again, as well as the risk parameter q_k. Eventually, we observe a period of relative "quietness" after the active period, which is followed again by an active period as soon as risk aversion decreases. In the top part of Fig. 3, for example, three such periods can be observe. Such an *intermittent dynamics* is also known from real financial markets, where it is called *volatility clustering* [7]. If the risk parameter q_k is not reduced fast enough, agents can very soon become bankrupt. But also the opposite could be true: agents can have such a low risk value, that they always fear losses and therefore do not invest even with a considerable budget. So, the dynamics of adjusting q_k plays a considerable role.

Finally, we note that the InvestorsGame presented in this paper can be easily extended towards more complex cases. For example, it would be interesting to allow different projects at the same time, which then may compete for the agents to invest. Another extension would be to replace bankrupt agents by new ones, which start with an initial budget. This in turn would mean an ongoing influx of new ressources, i.e. we have an open system instead of a closed one. So, we conclude that this simple MAS provides lots of possible scenarios for the study of economic and social problems.

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