Complex methods in economics: An example of behavioral heterogeneity in house prices

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Introduction

In this paper we estimate a Heterogeneous Agent Model (HAM) (Brock & Hommes, 1998) on house prices

- Financial crises and recessions are often preceded by decline in house prices (Reinhart & Rogoff, 2010)
- House prices show large swings around the fundamental prices
- Anecdotal: price expectations of most agents seem unreasonably positive during boom

Overall, there is suspicion of recurrent bubbles in house prices. A HAM might be able to capture this.
Introduction

Heterogeneous agent models (Brock & Hommes, 1997, 1998; Hommes, 2006)

- Developed to understand excess volatility in asset prices
- Brock & Hommes (1998): Asset price model with *boundedly rational agents*
  - Boundedly rational and heterogeneous *price expectations* (Frankel & Froot, 1991)
  - Agents learn and switch to short-term better-performing forecasting rules
  - Agents price expectations do not converge
- Asset prices display (unpredictable) bubbles and bursts
Introduction

In this paper we estimate a HAM (Brock & Hommes, 1998) on house prices in the United States and The Netherlands. This is a univariate nonlinear time series model. Objectives:

- Detect house price bubbles
- Anticipate decline in house prices
- Forecast house prices during boom and bursts
Housing market model

We adapt the asset price model of Boswijk, Hommes and Manzan (2007) to apply on housing markets.

Excess rate of return on housing

\[ R_{t+1} \equiv \frac{P_{t+1} - P_t + Q_{t+1}}{P_t} - r_t \]

\( \frac{P_{t+1} - P_t}{P_t} \) is capital gain, \( Q_{t+1} \) the cost of renting, and \( r_t \equiv r_t^{rf} + \omega_t \), the risk-free rate of return plus rate of maintenance costs.
Housing market model

Agent’s demand for housing units $z_{h,t}$ is determined by maximising risk-adjusted expected future excess returns, $R_{t+1}z_{h,t}$:

$$\max_{z_{h,t}} E_{h,t} (R_{t+1}z_{h,t}) - a \text{Var}_{h,t} (R_{t+1}z_{h,t})$$

We assume homogeneous expectations on the variance $\text{Var}_{h,t} (R_{t+1}) = V$

This gives the demand for type $h$

$$z_{h,t} = \frac{E_{h,t} (P_{t+1} + Q_{t+1})/P_t - (1 + r_t)}{aV}.$$
Define $n_{h,t}$ as the fraction of agents who have expectation rule $h$ in period $t$. ($\sum_{h=1}^{H} n_{h,t} = 1$). The equilibrium condition for market clearing of housing units in period $t$ is:

$$\sum_{h=1}^{H} n_{h,t} z_{h,t} = S,$$

where $S$ is the housing stock.

This leads to the price equation

$$P_t = \frac{1}{1 + r + \alpha} \sum_{h=1}^{H} n_{h,t} E_{h,t} (P_{t+1} + Q_{t+1}),$$

where $\alpha = aV \times S$.
Price equation

Define \( n_{h,t} \) as the fraction of agents who have expectation rule \( h \) in period \( t \). \((\sum_{h=1}^{H} n_{h,t} = 1)\). The equilibrium condition for market clearing of housing units in period \( t \) is:

\[
\sum_{h=1}^{H} n_{h,t} z_{h,t} = S,
\]

where \( S \) is the housing stock.

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P_t = \frac{1}{1 + r + \alpha} \sum_{h=1}^{H} n_{h,t} E_{h,t} (P_{t+1} + Q_{t+1}),
\]

where \( \alpha = aV \times S \)
Fundamental house price

We assume that rental costs $Q_t$ follow a geometric Brownian motion, such that

$$\frac{Q_{t+1}}{Q_t} = (1 + g)\varepsilon_{t+1},$$

with $g = e^{\mu + \frac{1}{2}\sigma^2} - 1$ and $\varepsilon_{t+1} = e^{\nu_{t+1} - \frac{1}{2}\sigma^2}$, s.t. $E_t(\varepsilon_{t+1}) = 1$.

Under rational expectations one obtains the fundamental price solution

$$P_t^* = \frac{1 + g}{r + \alpha - g} Q_t, \quad r + \alpha > g.$$
Fundamental house price

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Under rational expectations one obtains the fundamental price solution

$$P_t^* = \frac{1 + g}{r + \alpha - g}Q_t, \quad r + \alpha > g.$$
Deviation from the fundamental price

Upon introducing the *deviation from the fundamental* 

\[ X_t \equiv \frac{P_t - P_t^*}{P_t^*}, \]

the price equation can be rephrased as

\[ X_t = \frac{1}{\gamma} \sum_{h=1}^{H} n_{h,t} E_{h,t} (X_{t+1}), \quad \text{with} \quad \gamma = \frac{1+r+\alpha}{1+g}, \]

where we assume that the growth of the fundamental price is conditionally independent of \( \frac{P_{t+1}}{P_{t+1}^*} \).
Heterogeneous expectations

If all agents have rational expectations, then there is only one expectations rule that satisfies the price equation

\[ E_t[X_{t+1}] = 0 \]

However, we assume that agents are boundedly rational and have heterogeneous price expectations. We assume two types of expectation rules:

1. Fundamental-reverting: \( E_{1,t}[X_{t+1}] = \theta + \phi_1 X_{t-1}, \phi_1 < 1 \)
2. Fundamental-diverting: \( E_{2,t}[X_{t+1}] = \theta + \phi_2 X_{t-1}, \phi_2 > 1 \)
Heterogeneous expectations

The fraction of agents using expectations rule \( h \) is determined by the past performance \( \pi_{h,t-1} \) (realised profits) of the expectation rules

\[
\pi_{h,t-1} = (X_{t-1} - \gamma X_{t-2}) z_{h,t-2} = \text{cnst.} \times (X_{t-1} - \gamma X_{t-2})(\phi_1 X_{t-3} - \gamma X_{t-2})
\]

Fractions determined by logistic switching model

\[
n_{1,t} = \frac{\exp(\beta \pi_1, t-1)}{\exp(\beta \pi_1, t-1) + \exp(\beta \pi_2, t-1)}
\]

\[
n_{2,t} = 1 - n_{1,t}
\]
Estimation

Under the above expectation rules and market clearing conditions, deviations from fundamental house prices follow a nonlinear time series process.

Estimation via nonlinear LS regression

\[
SSE = \sum_{t=1}^{T} \left( X_t - \frac{\phi_1 n_{1,t} X_{t-1} + \phi_2 n_{2,t} X_{t-1}}{\gamma} \right)^2
\]

Note that estimation only requires a univariate time series of house prices, relative to its fundamental value.

Fundamental house prices are estimated by assuming a constant house price/rent ratio.
The US housing market

Estimated deviations from fundamental value, assuming a fundamental price-rent ratio

**price index and estimated fundamental price (left) and their relative deviation (right)**
Parameter estimates for US housing market

Estimated model parameters:

|        | Estimate | Std. Error | t value | Pr(>|t|)   |
|--------|----------|------------|---------|------------|
| $\phi_1$ | 0.892    | 0.059      | 15.071  | < 2e − 16 *** |
| $\phi_2$ | 1.130    | 0.069      | 16.308  | < 2e − 16 *** |
| $\beta$ | 2716     | 3463       | 0.784   | 0.434      |
| $\theta$ | 0.0012   | 0.0009     | 1.318   | 0.189      |
| $\gamma$ | 1.010    | 0.015      | 68.453  | < 2e − 16 *** |

Signif. codes: 0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 .’ 0.1 ’ ’ 1

Residual standard error: 0.01104 (156 degrees of freedom)

$$\left| \frac{\hat{\phi}_1 + \hat{\phi}_2}{2\hat{\gamma}} \right| = 1.0010$$
Estimated time-dependent fractions (US)

\[ \text{ts.union}(y_t, n_1, a_{r1}) \]

![Time series plot showing estimated time-dependent fractions (US)](image)
Fancharts US

USA - BHM insample forecasts

USA - BHM out of sample forecasts

USA - AR in sample forecasts

USA - AR out of sample
The Dutch housing market

Estimated deviations from fundamental value, assuming a fundamental price-rent ratio

price index and estimated fundamental price (left) and their relative deviation (right)
Parameter estimates for NL housing market

Estimation of $\gamma$ leads to unrealistically large values (1.51) $\Rightarrow \gamma$
fixed at 1.01

Estimated model parameters:

|   | Estimate | Std. Error | $t$ value | Pr(>|$t$|) |
|---|----------|------------|-----------|-----------|
| $\phi_1$ | 0.9849   | 0.01495    | 65.865    | $2 \times 10^{-16}$ *** |
| $\phi_2$ | 1.040    | 0.01576    | 65.996    | $2 \times 10^{-16}$ *** |
| $\beta$  | 12420    | 21050      | 0.590     | 0.556     |
| $\theta$ | 0.00299  | 0.002071   | 1.444     | 0.151     |
| $\gamma$ | 1.01     | —          | —         | —         |

Signif. codes: 0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1

Residual standard error: 0.0259 (157 degrees of freedom)

$$\left| \frac{\hat{\phi}_1 + \hat{\phi}_2}{2\hat{\gamma}} \right| = 1.0024$$
Estimated time-dependent fractions (NL)

\[ \text{ts.union}(y_1, n_1, a_1) \]

- Time series of \( y_1 \), \( n_1 \), and \( a_1 \) from 1970 to 2010.

Graph showing the time series for \( y_1 \), \( n_1 \), and \( a_1 \) over the years 1970 to 2010.
Summary/conclusions

- Housing market model with heterogeneous price expectations developed and estimated empirically
- Agents in USA switched to fundamental-diverting forecasting rule in 2004-2005
  - House bubble!
- Forecasts from HAM differ strongly from linear AR model
Future work

The presented work should be seen as a starting point of a long-term project to develop an alternative for SVAR and DSGE models.

Future work

- Develop a full housing market model with housing search
- Alternative expectation rules (based on experimental evidence)
- Out-of-sample performance