Interpreting Directed Clustering Coefficients in Complex Banking Networks

B. M. Tabak\textsuperscript{1}  M. Takami\textsuperscript{1}  J. Rocha\textsuperscript{1}  D. Cajueiro\textsuperscript{1}
S. Souza\textsuperscript{1}

\textsuperscript{1}Central Bank of Brazil

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Can Economics, as a scientific discipline that must extricate itself from its current conceptual crisis, benefit from concepts, methods and insights developed in other disciplines, notably, the natural sciences?

To answer to this question, it is necessary to find characteristics that both classes of sciences have in common.

One action to be taken is to decompose the object studied by Economics into elements. For instance, an interaction between:

- **Nature / Production**
  Includes natural resources and the production chain: these are subject to natural laws

- **Monetary System / Allocation**
  This element is also subject to constraints that apply to stocks and flows, but there are some particularities, for instance, the money creation when a bank grants credit to a depositor.
Economics & Natural Sciences

- Human decisions
  This element adds complexity to the economic problem: they aren’t deterministic; moreover, they are subject to non-observable factors or externalities that can change completely the outcome of the system.

- In usual conditions, human behaviour follows (loosely) rules. This can be seen from our daily life.

- Specifically, social and individual decisions are shaped by formal and informal institutions, which are defined by North (1990) as being the rules of the game.

- But, during crises... these rules may change.
We propose that

- An economy can be modelled as a complex system. If human decisions can’t be modelled, it must be possible to build models of the other elements conditional to some sets of human decisions/policies.
- The idea is that the economy has a “mechanical” component which behavior imposes limits on the results that can be achieved due to a decision.
- Applications: ABMs, multi-level networks, ...
This work

Is an attempt of using directed clustering coefficients to interpret phenomena in banking networks.

- One problem in Economics is to find measures that represent the state of some attribute of the economy. There is a great number of agents interacting, which sometimes makes difficult to identify or measure, at least qualitatively, what is going on.

- In this paper, we use directed clustering coefficients, as defined in Fagiolo (2007), to identify the roles of banks in financial systems.

- We also use them to identify possible sources of systemic risk.
Banking lending networks

- Banking lending networks are one of the most important financial systems.
- In a banking system, there are different types of banks, defined from the point of view of their business lines, or size, or ownership.
- These bank types may be associated with specific roles in the banking network and, also, with the possibility of being a source of risk.
- We use directed clustering coefficients to uncover this information.
Directed Clustering Coefficients - Notation

\( A \): directed adjacency matrix of the network

\( W \): directed matrix of weights that represents the network.

\( d_i^{\text{in}} \): in-degree of node \( i \)

\( d_i^{\text{out}} \): out-degree of node \( i \)

\( d_i^{\text{tot}} = d_i^{\text{in}} + d_i^{\text{out}} \): total degree of node \( i \).

\( d^{\leftrightarrow} = \sum_{j \neq i} a_{ij} a_{ji} = A_{ii}^2 \): number of “two-way” degrees.
In binary directed networks, the clustering coefficient of node \( i \) for a binary network may be defined as the ratio between all the existing triangles formed by \( i \) and 2 of its neighbors, and the number of all possible triangles that could be formed:

\[
C_i^D(A) = \frac{(A + A^T)_{ii}^3}{2[d_i^{tot}(d_i^{tot} - 1) - 2d_i^{\leftrightarrow}]}.
\]
Clustering Coefficient - Weighted Directed Networks

The clustering coefficient defined for the unweighed case is extended to the weighted case by replacing the number of existing directed triangles formed with its weighted counterpart:

\[ \tilde{C}_i^D(W) = \frac{[\hat{W} + (\hat{W}^T)]_{ii}^3}{2[d_i^{tot}(d_i^{tot} - 1) - 2d_i^{\leftrightarrow}]} \]

where \( \hat{W} = W^{[\frac{1}{3}]} = [w_{ij}^{\frac{1}{3}}] \).
Definitions

- Loosely speaking, these coefficients measure the extent at which a node is involved in triangular relationships with its neighbors.
- However, these two definitions are not enough to characterize the richness of patterns that take place in a complex directed network.
- In fact, these equations treat all the possible triangles as if they had the same pattern.
- However, in directed graphs, edges that point in different directions should be interpreted differently.
- Therefore, a specific clustering coefficient is defined for each of these patterns of directions.
Patterns - Cycle Clustering

Directed Clustering Coefficients

Building an Interpretation

Application and Results

Sergio R. Souza

Directed Clustering Coefficients in Banking Networks
Middleman Clustering

\[ h \]

\[ i \quad j \]

\[ h \]

\[ i \quad j \]
In Clustering
Out Clustering

\[ \text{Directed Clustering Coefficients} \]

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Directed Clustering Coefficients in Banking Networks
Context

We consider that one of the main functions of a bank into the economic system is to provide liquidity to non-banks, transforming maturities when necessary.

So, we see interbank market transactions as consequences of needs or excess of liquidity banks may have when performing their core business.
Roles of banks in the network

Taking into account that banks in the network have relationships outside the network, their clustering coefficients count their relational positions as follows:

- In a non-cyclical triangular relationship, there exists an *in*, an *out* and a *middleman* bank.
- The *in* bank is borrowing from other banks to lend and investing the funds outside the network.
- The *out* bank is raising funds outside (demand or term deposits) to invest in the network.
- The *middleman* bank is transforming maturities, borrowing in the short term to lend in the long, or doing both things above.
- In a cyclical triangular relationship, if one performs a netting, the lower weight edge disappears, reducing the amount remaining.
Aggregation of triangles for each bank

- We don’t understand that the each triangle is representative of real transactions. In fact, what really is happening is that the bank’s front-office is doing transactions outside the network and generating cashflow needs at different dates. These generates needs or excess of funds, which are driven to the interbank market.

- The idea is that the aggregation of a bank’s triangles may be associated with an aggregation of transactions of the associated type.

- This is not the only way to uncover the roles banks play, but it doesn’t use balance sheet information, only bilateral exposure data.
Clustering Coefficients and Risk

- We use the weighted clustering coefficients
- Individually taken, the *in* and *out* coefficients are associated with an interbank borrowing / lending position
- But there is a share of the risk which is not represented in these coefficients:

![Diagram showing a zero clustering coefficient network]

**Figure:** A zero clustering coefficient network
So

- If the weights of edges are exposures scaled by the total debt in the network, one have in each edge the fraction of this amount.

- So, if values of the clustering coefficients of a bank strongly exceed the mean debt of the banks, they will be indicative of risk.

- If $\tilde{C}_i^{\text{in}}$ is high, the bank $i$ is a source of risk to its counterparties;

- If $\tilde{C}_i^{\text{out}}$ is high, (other coefficients low), the bank is taking significant risk from the system.
Data

- Brazilian interbank market - exposures data
- Exposures between banks - Interbank and fixed income.
- Daily exposures for a large time series (Jan 2004 - Nov 2007) - large high frequency panel data!
- All banks operating within the Brazilian economy, by ownership category - public, domestic and foreign.
Mean $\tilde{C}_i^{in}$ by bank category

Figure: In weighted CC. Solid: State, dotted: Foreign, dashed: Domestic
Mean $\tilde{C}_i^{out}$ by bank category

**Figure:** Out weighted CC. Solid: State, dotted: Foreign, dashed: Domestic
Results

- We have evaluated these clustering coefficients for the Brazilian interbank market: they indicate public banks, in general borrow from the network to lend outside. The other banks don’t show such a defined behavior.

- The level of risk inferred from the magnitudes is low.

- Furthermore, we find that $\tilde{C}_i^{mid}$ they are negatively correlated with interesting rate - which imply that credit constraints (increase of interest rate) reduce their intermediation activity.
Conclusions

- We can employ clustering coefficients to identify roles banks play in the interbank network.
- These coefficients underestimate risk.
- It is possible to use these coefficients as identifiers of risky institutions, but: they underestimate risk and need further improvements.
Any suggestions or exchange of ideas -
benjaminm.tabak@gmail.com or sergio.souza@bcb.gov.br.
Thanks for the attention!