Michel Barnier, the EU’s financial services chief, has proposed that bank investors should have set maximum ratios on the size of their bonuses compared with their fixed pay.

Bonuses that are a “large” multiple of fixed pay “are likely to encourage excessive risk taking and undermine confidence in the financial sector generally,” according to the plans.
Contexts

• Decision Theory
• Certainty
• Risk
• Uncertainty

• Game Theory- Strategic

Knight, 1921; Luce and Raiffa, 1957.
Decision making

- Decision theory (Certainty, Risk, Uncertainty)

- Risk- Single DM and risky prospects (well defined-option set, probability space, and payoffs)

- Typically very simple gambles are used to measure people’s preferences for risk
Risky decision making

- Purview of Decision Theory
- Static risky decision
  - A: a sure gain of 240
  - B: a 25% chance to gain 1000 (75% chance of nothing)

Common tool in EUT, Prospect theory
Risky decision making

- Purview of Decision Theory

- Static risky decision
  - A: a sure gain of 240
  - B: a 25% chance to gain 1000 (75% chance of nothing)

Behavioral tendency to prefer the sure thing; risk aversion

Common tool in EUT, Prospect theory
Risky decision making

- Good draws are worth 1
- Bad draws result in bankruptcy and the termination of the task
- Draws are made with replacement
- The DM may make one draw at a time
- The choice for the DM is when to stop making draws

90% good
10% bad
Dynamic risky decision making

- Devil’s Task (Slovic, 1966)
- Iowa Card Task (Bechara et al., 1994)
- Balloon Analog Risk Task- BART (Lejuez et al., 2002)
- Columbia Card Task (Figner et al., 2009)

See Edwards (1962)
Dynamic risky decision making

- Devil’s Task (Slovic, 1966)
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Other simple dynamic risky and uncertain multi-stage decision tasks

See Edwards (1962)
Dynamic risky decision making

- Devil’s Task (Slovic, 1966)
- Iowa Card Task (Bechara et al., 1994)
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- Devil’s Task (Slovic, 1966)
- Iowa Card Task (Bechara et al., 1994)
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- Columbia Card Task (Figner et al., 2009)

Dynamic, well defined payoffs and risks, constant risk level

See Edwards (1962)
Sequential draw task

- Good draws are worth \( v = 1 \)
- Bad draws result in bankruptcy (i.e. a payoff of 0) and the termination of the task
- Draws are made with replacement
- Probabilities- Well defined and stable

\[
\begin{align*}
p_w &= 0.9 \\
p_l &= (1-p_w) = 0.1
\end{align*}
\]
Sequential draw task

- What is the normative solution to this task?

- At each stage, the DM is choosing between: a sure payoff of their current holdings ($h$) vs. the risky option to marginally increase their holdings by $v$ with probability $p$.

\[ p_w = 0.9 \]
\[ p_l = (1-p_w) = 0.1 \]
\[ v = 1 \]
Sequential draw task

- What is the normative solution to this task?

- At each stage, the DM is choosing between: a sure payoff of their current holdings \((h)\) vs. the risky option to marginally increase their holdings by \(v\) with probability \(p\).

\[
h \leq p_w \cdot (h + v)
\]

\[
h^* = \frac{p \cdot v}{1 - p}
\]
Optimal number of draws

- \( p_w = 0.95 \) \( h^* = 19 \)
- \( p_w = 0.9 \) \( h^* = 9 \)
- \( p_w = 0.8 \) \( h^* = 4 \)
The task is sensitive to individual differences in risk aversion.
More than one decision maker

One decision maker

Decision Theory
Certainty
Risk
Uncertainty

Game Theory - Strategic

More than one decision maker

Knight, 1921; Luce and Raiffa, 1957.
The players make their draws simultaneously and privately. The player with the most points wins the game and has a payoff of 1. The loser earns nothing. Ties are broken randomly. All of this information is common knowledge.

See Shapley (1953)
• What is the normative solution to this game?
• $EV_{\text{max}}$ does not help...
• If player 1 aims for 9 points, player 2 can beat him 58% of the time by only aiming for 1 point.
• If player 1 realizes this, he can aim for 2 points and then win 78% of the time
• If player 2 realizes this...

A very simple stochastic game

Player 1
$p_w = 0.9$
$p_l = (1-p_w) = 0.1$
$v = 1$ point

Player 2
$p_w = 0.9$
$p_l = (1-p_w) = 0.1$
$v = 1$ point

• Simultaneous and private draws
• The most points wins
• Ties broken randomly
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Constant sum game, cells show the p(win) and EV for Player 1 given a strategy profile.
A constant sum game is depicted, where the cells show the probability of winning (p(win)) and the expected value (EV) for Player 1 given a strategy profile. The game matrix is shown with Player 1 on the left and Player 2 on the top. The values in the cells indicate the outcomes for Player 1, with higher values typically indicating a greater chance of winning or a higher expected value.
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Constant sum game, cells show the p(win) and EV for Player 1 given a strategy profile.
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Strategy of 12 (or greater) is strictly dominated by a mix of strategies 1 to 11
Mixed strategy equilibrium

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Pure strategy NE does not exist. This is the unique symmetric mixed strategy equilibrium.
Mixed strategy equilibrium

- Simultaneous and private draws
- The most points wins
- Ties broken randomly

Pure strategy NE does not exist. This is the unique symmetric mixed strategy equilibrium.
Unique mixed strategy equilibrium

$p = 0.9000$

![Graph showing a bar chart with the x-axis labeled 'Strategy in number of draws' and the y-axis labeled 'Play with probability'. The chart indicates a distribution where the strategy with the highest probability is around 10 draws.]
Discontinuity in the NE as $p(\text{win})$ changes
A very simple stochastic non-constant sum game

Player 1

\[ p_w = 0.9 \]
\[ p_l = (1 - p_w) = 0.1 \]
\[ v = 1 \]

Player 2

\[ p_w = 0.9 \]
\[ p_l = (1 - p_w) = 0.1 \]
\[ v = 1 \]

The players make their draws simultaneously and privately.
The players can keep their earnings from draws.
The player with the most earnings from draws also earns a bonus of 1.
The loser earns no bonus. Ties are broken randomly.
All of this information is common knowledge.
A very simple stochastic non-constant sum game

- Payoffs can come from two sources: retained draws, and winning the bonus.
- What is the normative solution to this game?
- To maximize payoffs from draws alone, a DM should draw 9 times. But this is not an equilibrium strategy.

Player 1
- Simultaneous and private draws
- The most points wins bonus
- Ties broken randomly
- Retained draws still yield a payoff

\[ p_w = 0.9 \]
\[ p_l = (1-p_w) = 0.1 \]
\[ v = 1 \text{ point} \]

Player 2
- \[ p_w = 0.9 \]
- \[ p_l = (1-p_w) = 0.1 \]
- \[ v = 1 \text{ point} \]
A very simple stochastic non-constant sum game

\[ p_w = 0.9 \]
\[ p_i = (1-p_w) = 0.1 \]
\[ v = 1 \text{ point} \]

\[ p_w = 0.9 \]
\[ p_i = (1-p_w) = 0.1 \]
\[ v = 1 \text{ point} \]

**Player 1**

- Simultaneous and private draws
- The most points wins bonus
- Ties broken randomly
- Retained draws still yield a payoff

**Player 2**

- Payoffs can come from two sources - retained draws, and winning the bonus.
- What is the normative solution to this game?
- To maximize payoffs from draws alone, a DM should draw 9 times. But this is not an equilibrium strategy.

**Mixed strategy equilibrium**

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<td>0.177</td>
<td>0.784</td>
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</table>
• Payoffs can come from two sources—retained draws, and winning the bonus.

• What is the normative solution to this game?

• To maximize payoffs from draws alone, a DM should draw 9 times. But this is not an equilibrium strategy.

Mixed strategy equilibrium

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A very simple stochastic non-constant sum game

- Simultaneous and private draws
- The most points wins bonus
- Ties broken randomly
- Retained draws still yield a payoff
• This stochastic game with payoffs from both returns on risk and the bonus is a social dilemma
• These are instances where individual rationality leads to collective demise
• In this game the bonus induces both of the players to take larger risks than are optimal in the non-strategic case
• As the bonus increases, the collective efficiency decreases and players are induced to play more varied strategies

Mixed strategy equilibrium

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A very simple stochastic non-constant sum game
• As the bonus increases, the collective efficiency decreases and players are induced to play more varied strategies

• The equilibrium strategy here sacrifices 7.3% of potential earnings

• Offering a bonus to rational players requires spending more money that induces players to take non-optimal risks, play a more varied set of strategies, and ultimately all make less money

<table>
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<tr>
<th>Mixed strategy equilibrium</th>
<th>(10:1 ratio between bonus and risk payoff)</th>
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More than one decision maker

One decision maker

Decision Theory
Certainty
Risk
Uncertainty

Game Theory - Strategic
Interdisciplinary economics

- Can economics as a scientific discipline that must extricate itself from its current conceptual crisis, benefit from concepts, methods and insights developed in other disciplines, notably the natural sciences?

- Computational methods

- Behavioral and cognitive considerations vs. remarkably delicate equilibrium

- Better anticipate unexpected consequences from different incentive structures

- i.e. Bonuses can induce rational but inefficient behavior
Simple Stochastic Games: Risk Taking in Strategic Contexts

Ryan O. Murphy

Chair of Decision Theory and Behavioral Game Theory
ETH Zürich

Latsis Symposium  September 12, 2012

www.dbgt.ethz.ch  rmurphy@ethz.ch