Quantifying reflexivity in financial markets: towards a prediction of flash crashes

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based on joint works with D. Sornette (ETH Zurich), D. Bicchetti and N. Maystre (UNCTAD)
How do financial markets work?
**Efficient Market Hypothesis**  
(exogenous dynamics)

Prices are just reflecting news: the market fully and instantaneously absorbs the flow of information and faithfully reflects it in asset prices. In particular, financial crashes are the signature of exogenous negative news of large impact.

**“Reflexivity” of markets**  
(endogenous dynamics)

Markets are subjected to internal feedback loops (e.g. created by collective behavior such as herding or informational cascades).

Prices do influence the fundamentals and this newly-influenced set of fundamentals then proceed to change expectations, thus influencing prices.
Empirical findings about market prices

**Volatility clustering and long memory in volatility**

Time reversal asymmetry (e.g. “leverage” effect)

Another stylized facts:  
- Multifractality  
- Heavy-tailed distributions  
- Asymmetry in time scales  
- Gain / loss asymmetry, etc.
First direct evidences

“Excess volatility” puzzle

Real S&P 500 value

-----  *Ex post* rational value

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“What moves stock prices?”

<table>
<thead>
<tr>
<th>Date</th>
<th>Percent Change</th>
<th>New York Times Explanation*</th>
</tr>
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<tbody>
<tr>
<td>1 Oct. 19, 1987</td>
<td>~20.47</td>
<td>Worry over dollar decline and trade deficit; <em>fear of US not supporting dollar</em>.</td>
</tr>
<tr>
<td>2 Oct. 21, 1987</td>
<td>9.10</td>
<td>Interest rates continue to fall; deficit talks in Washington; bargain hunting.</td>
</tr>
<tr>
<td>3 Oct. 26, 1987</td>
<td>~8.28</td>
<td>Fear of budget deficits; margin calls; reaction to falling foreign stocks.</td>
</tr>
<tr>
<td>4 Sep. 3, 1946</td>
<td>~6.73</td>
<td>“No basic reason for the assault on prices.”</td>
</tr>
<tr>
<td>8 Oct. 20, 1987</td>
<td>5.33</td>
<td>Investors looking for “quality stocks.”</td>
</tr>
<tr>
<td>9 Sep. 9, 1946</td>
<td>~5.24</td>
<td>Labor unrest in maritime and trucking industries.</td>
</tr>
<tr>
<td>10 Oct. 16, 1987</td>
<td>~5.16</td>
<td>Fear of trade deficit; fear of higher interest rates; tension with Iran.</td>
</tr>
<tr>
<td>11 May 27, 1970</td>
<td>5.02</td>
<td>Rumors of change in economic policy. “The stock surge happened for no fundamental reason.”</td>
</tr>
<tr>
<td>12 Sep. 11, 1986</td>
<td>~4.81</td>
<td>Foreign governments refuse to lower interest rates; crackdown on triple witching announced.</td>
</tr>
<tr>
<td>13 Aug. 17, 1982</td>
<td>4.76</td>
<td>Interest rates decline.</td>
</tr>
</tbody>
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“Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?”


D. Cutler, J. Poterba, L. Summers (1987)

“What moves stock prices?”

Probability of having **jump** in price at time $t$, conditional on:
- (red) **news** at time $t=0$
- (black) **jump** at time $t=0$

Epidemics in socio-economic networks

\[ A(t) \sim 1/t^{1-\theta} \]

\[ A(t) \sim 1/t^{1-2\theta} \]

Thy Neighbor's Portfolio: Word-of-Mouth Effects in the Holdings and Trades of Money Managers

HARRISON HONG, JEFFREY D. KUBIK, and JEREMY C. STEIN

ABSTRACT

A mutual fund manager is more likely to buy (or sell) a particular stock in any quarter if other managers in the same city are buying (or selling) that same stock. This pattern shows up even when the fund manager and the stock in question are located far apart, so it is distinct from anything having to do with local preference. The evidence can be interpreted in terms of an epidemic model in which investors spread information about stocks to one another by word of mouth.

In this paper, we explore the hypothesis that investors spread information and ideas about stocks to one another directly, through word-of-mouth communication. This hypothesis comes up frequently in informal accounts of the behavior of the stock market. For example, in his bestseller *Irrational Exuberance*, Shiller (2000) devotes an entire chapter to the subject of “Herd Behavior and Epidemics,” and writes

A fundamental observation about human society is that people who communicate regularly with one another think similarly. There is at any place and in any time a Zeitgeist, a spirit of the times... Word-of-mouth transmission of ideas appears to be an important contributor to day-to-day or hour-to-hour stock market fluctuations. (pp. 148, 155)
Sources of reflexivity

- Behavioral mechanisms such as imitation and informational cascades leading to herding;
- Speculation, based on technical analysis, including algorithmic trading;
- **Hedging** strategies (also increase cross-excitation between markets);
- Methods of **optimal portfolio execution** and order splitting;
- Margin/leverage trading and **margin-calls**;
- **Stop-loss orders** and etc;
- **High frequency trading (HFT)** as a subset of algorithmic trading.
How efficient is the real market?

Is it possible to quantify the interplay between exogeneity and endogeneity?

Can we develop diagnostics of fragility and of incoming crises as well as upside potentials?
“As a policy-maker during the crisis, I found the available models of limited help. In fact, I would go further: in the face of the crisis, we felt abandoned by conventional tools. In the absence of clear guidance from existing analytical frameworks, policy-makers had to place particular reliance on our experience”.

Jean-Claude Trichet (2010)
Inspiration from geophysics

The best of the best model for modeling and forecasting of activity is the **Epidemic-Type Aftershock Sequence (ETAS)**, that belongs to the class of **self-excited conditional Poisson (Hawkes) processes**:

\[
\lambda(t) = \mu + \sum_{t_i < t} \phi(t - t_i; m_i; \vec{r} - \vec{r}_i)
\]
Crucial parameter of the branching process is the “branching ratio” \( n \) - average number of “daughters” per one “mother”

For \( n < 1 \) system is **subcritical** (stationary evolution)
For \( n = 1 \) system is **critical** (tipping point)
For \( n > 1 \) system is **supercritical** (with prob.>0 will explode to infinity)

In subcritical regime, the branching ratio \( (n) \) is equal to the fraction of *endogenously generated events* among the whole population.
Self-excited Hawkes process is the point process whose intensity $\lambda_t(t)$ is conditional on its history:

$$\lambda(t) = \mu + n \sum_{t_i < t} \varphi(t - t_i)$$

Financial applications of Hawkes model:

- High-frequency price dynamics
- Order book construction
- Critical events and estimation of VaR
- Correlated default times in a portfolio of companies
E-mini S&P 500 Futures Contract (ES@CME):

- Trading time: 23.25 hours/day (active trading: 6.75 hours)
- Contract size: $50 x E-mini S&P 500 futures price
- Tick size: 0.25 index points=$12.50
- **Average daily volume in 2010: 2,194,975** (for comparison: average daily volume of regular S&P 500 futures: 345,483)
Estimation of short-term reflexivity

Trading activity
proxied by volume and number of mid-price changes

Dynamics of price and volatility

Rate of exogenous events (triggered by idiosyncratic “news”)

Branching ratio that quantifies reflexivity of the system
(fraction of endogenous events in the system)

“Flash-crash” of May 6, 2010

\[\text{Graph showing market events of May 6, 2010} \]

- "Findings Regarding the Market Events of May 6, 2010": Report of the Staffs of the CFTC and SEC to the Joint Advisory Committee on Emerging Regulatory Issues (September 30, 2010).
Exogenous vs endogenous shocks in HF

April 27, 2010:
Significant fall of most of US markets following the cut of the credit rating of Greece and Portugal

May 6, 2010 (“flash-crash”):
The activity of high-frequency traders of the S&P 500 E-mini futures contracts leaded to a dramatic fall in other markets

Volume and Trading activity behave similar in both cases

Branching ratio (degree of reflexivity) reveals fundamental difference between two shocks

Final remarks

- In contrast to “neo-classical” theories, news play a minor role in price dynamics.

- Markets are operating close to criticality, implying significant role of endogenous feedback mechanisms in price formation process.

- Over the past decade the role of short-term reflexivity (attributed mostly to AT and HFT) grew up enormously.

- The class of self-excited models that combines (i) external influence on system with (ii) feedback mechanisms provides a novel powerful metric of endogeneity, which is much richer than standard direct measures of activity such as volume and trading rates.

- This measure based on the branching ratio allows real-time diagnostics of the market and distinguishing of exogenous (triggered by news) and endogenous (self-excited) events.