Channels of contagion: identifying and monitoring systemic risk

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Outline

• Mechanisms for contagion in banking systems
• Banking systems as counterparty networks
• A case study: the Brazilian banking system
• Default contagion on financial networks: insolvency cascades and illiquidity cascades.
• The importance of metrics when assessing contagion
• An indicator for systemic importance: the Contagion Index.
• Resilience to contagion in financial networks: some analytical results for networks with arbitrary topology
• What makes a network vulnerable to contagion?
• A simulation-free approach to systemic stress testing
Systemic Risk

• Systemic risk may be defined as the risk that a significant portion of the financial system fails to function properly.
• The monitoring and management of systemic risk has become a major issue for regulators and market participants since the 2008 crisis.
• The financial crisis has simultaneously underlined
  · the importance of contagion effects and systemic risk
  · the lack of adequate indicators for monitoring systemic risk.
  · the lack of adequate data for computing such indicators
Many initiatives under way: creation of derivatives clearinghouses, legislation on transparency in OTC markets, creation of Office of Financial Research (US), various Financial Stability Boards
BUT: methodological shortcomings, open questions
Systemic Risk: mechanisms

- Why do many financial institutions simultaneously default or suffer large losses?

1. **Concentration**: exposure of a large number of institutions to a common risk factor

2. **Balance sheet contagion**: the default of one institution may lead to writedowns of assets held by its counterparties which may result in their insolvency.

3. **Spirals of illiquidity**: market moves and/or credit events may lead to margin calls/short term liabilities which lead to default of institutions which lack sufficient short term funds.

4. **Price-mediated contagion**: fire sales of assets due to deleveraging can further depreciate asset prices and lead to losses in other portfolios, generating feedback and instability.

- The default of Amaranth hardly made headlines: no systemic impact.
- The default of LTCM threatened the stability of the US banking system → Fed intervention
- Reason: LTCM had many counterparties in the world banking system, with large liabilities/exposures.
- 1: Systemic impact is not about ’net’ size but related to exposures/ connections with other institutions.
- 2: a firm can have a small magnitude of losses/gains AND be a source of large systemic risk
Systemic Risk

Various questions:

Mechanisms which lead to systemic risk

Metrics for systemic risk

Monitoring of systemic risk: data type/granularity?

Management and control of systemic risk by regulators

Need for quantitative approaches to these questions

Mathematical / Quantitative Modeling can and should play a more important role in the study of systemic risk and in the current regulatory debate.
Understanding systemic risk: can ideas from science and engineering help?

• The financial system may be modeled as a network – a weighted, directed graph- whose nodes are financial institutions and whose links represents exposures and receivables.
• Cascades of insolvency and illiquidity may be modeled as contagion processes on such networks
• -> useful analogies with epidemiology, stability of power grids, security of computer networks, random graph theory, percolation theory
• BUT: Data on interbank exposures reveal a complex, heterogeneous structure which is poorly represented by simple network models used in the theoretical literature.
The network approach to contagion modeling

A financial system is naturally modeled as a network of counterparty relations: a set of nodes and weighted links where

- nodes $i \in V$ represent financial market participants: banks, funds, corporate borrowers/lenders, hedge funds, monolines.
- (directed) links represent counterparty exposures: $E_{ij}$ is the exposure of $i$ to $j$, the maximal loss of $i$ if $j$ defaults
- $E_{ij}$ is understood as (positive part) of the market value of contracts of $i$ with $j$.
- $L_{ij} = E_{ji}$ is the total liability of $i$ towards $j$.
- Each institution $i$ disposes of
  - a capital $c_i$ for absorbing market losses. Proxy for $c_i$: Tier I capital.
- a liquidity buffer $l_i$

<table>
<thead>
<tr>
<th>Assets $A_i$</th>
<th>Liabilities $L_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interbank assets</td>
<td>Interbank liabilities</td>
</tr>
<tr>
<td>$\sum_j E_{ij}$</td>
<td>$\sum_j E_{ji}$</td>
</tr>
<tr>
<td>including: Liquid assets</td>
<td>Deposits</td>
</tr>
<tr>
<td>$l_i^0$</td>
<td>$c_i$</td>
</tr>
<tr>
<td>Other assets $a_i$</td>
<td>Capital</td>
</tr>
</tbody>
</table>

Table 1: Stylized balance sheet of a bank.
- Capital absorbs first losses.

- Default occurs if
  - (i) Demand for immediate payments (margin calls, derivative payouts) exceeds liquidity: \( l_i + \sum_{j \neq i} \pi_{ij} < 0 \)
    
    Requires monitoring liquidity reserves and tracking potential future exposures/payouts from derivatives.

  - (ii) Loss due to counterparty exposure \( > c_i \) ⇒ “insolvency”
    
    ⇒ lenders cut off short term funding ⇒ (i)

- Actual, not (Basel-type) “risk-weighted” value of exposures, assets and liabilities need to be considered.
Measuring the systemic impact of a default

Objective: quantify the losses generated across the network by the initial default of a given financial institution.

Defaults can occur through

1. (correlated) market shocks to balance sheets
   \[ c_i \mapsto \max(c_i + \epsilon_i, 0) \]

2. counterparty risk: default of \( i \) may lead to default of \( j \) if
   \[ c_j < E_{ji} \]

3. lack of liquidity: if margin calls/ derivative payouts \( \pi_{ij} \) exceed available liquidity
   \[ l_i + \sum_j \pi_{ij}(c + \epsilon, E) < 0 \]

In cases 2 and 3 this can generate a 'domino effect' and initiate a cascade of defaults.
Figure: Network structures of real-world banking systems. Austria: scale-free structure (Boss et al. 2004), Switzerland: sparse and centralized structure (Müller 2006).
Figure: Network structures of real-world banking systems. Hungary: multiple money center structure (Lubloy et al 2006) Brazil: scale-free structure (Cont, Bastos, Moussa 2010).
The Brazil financial system: a directed scale-free network

- Exposures are reported daily to Brazilian central bank.
- Data set of all consolidated interbank exposures (incl. swaps) + Tier I and Tier II capital (2007-08).
- $n \approx 100$ holdings/conglomerates, $\approx 1000$ counterparty relations
- Average number of counterparties (degree) = 7
- Heterogeneity of connectivity: in-degree (number of debtors) and out-degree (number of creditors) have heavy tailed distributions
  \[
  \frac{1}{n} \#\{v, \text{indeg}(v) = k\} \sim \frac{C}{k^{\alpha_{in}}} \quad \frac{1}{n} \#\{v, \text{outdeg}(v) = k\} \sim \frac{C}{k^{\alpha_{out}}}
  \]
  with exponents $\alpha_{in}, \alpha_{out}$ between 2 and 3.
- Heterogeneity of exposures: heavy tailed Pareto distribution with exponent between 2 and 3.
Figure 3: Brazilian financial network: distribution of in-degree.
Figure 4: Brazilian financial network: stability of degree distributions across dates.
Figure 6: Brazilian network: distribution of exposures in BRL.
Domino effects in financial networks: empirical studies

Empirical studies on interbank networks by central banks:


examine by simulation the impact of single or multiple defaults on bank solvency in absence of other effects (e.g. market shocks).

Mostly focused on payment systems (FedWire) or FedFunds exposures and report very small loss magnitudes (in % of total assets).
Metrics for systemic importance

- Objective: identify institutions or groups of institutions whose failure threatens the stability of the financial system
- Quantify the potential loss due to such a default -> identification of SIFIs, basis for systemic tax, etc.

The majority of previous empirical/simulation studies has systematically underestimated the magnitude of contagion effects due to the use of

- indicators based on idiosyncratic default which try to isolate contagion from market risk: i.e. scenarios where one bank fails in an isolated way while everything else remains equal. In fact most large bank defaults happen in macroeconomic stress scenarios
- indicators based on averaging (across scenarios, across banks): while default of most banks have little systemic impact, some have large systemic impact
  -> heavy tailed cross sectional distributions, average is a poor statistic

- Price-based (as opposed to exposure-based) indicators: backward looking instead of forward looking, no predictive power
Default cascades and default impact

Given an initial set $A$ of defaults in the network, we define a sequence $D_k(A)$ of default events by setting $D_0(A) = A$ and, at each iteration, identifying the set $D_k(A)$ of institutions which either

- become *insolvent* due to their counterparty exposures to institutions in $D_{k-1}(A)$ which have already defaulted at the previous round

\[
c_j^k = \min(c_{j}^{k-1} - \sum_{v \in D_{k-1}(A)} (1 - R_v)E_{jv}, 0)
\]  
(7)

- lack the *liquidity* to pay out the contingent cash flows $\pi_{jv}(c^{k-1}, E^{k-1})$–margin calls or derivatives payables– triggered by the previous credit/market events:

\[
l_j + \sum_{v \notin D_{k-1}(A)} \pi_{jv}(c^{k-1}, E^{k-1}) < 0
\]
(8)
**Definition 1** (Default cascade). *Given the initial default of a set $A$ of institutions in the network, the default cascade generated by $A$ is defined as the sequence

$$D_0(A) \subset D_1(A) \subset \cdots \subset D_{n-1}(A)$$

where $D_0(A) = A$ and for $k \geq 1$,

$$c_j^k = \min(c_j^{k-1} - \sum_{v \in D_{k-1}(A)} (1 - R_v)E_{jv}, 0) \quad (9)$$

$$D_k(A) = \{ j, \quad c_j^k = 0 \text{ or } l_j + \sum_{v \notin D_{k-1}(A)} \pi_{jv}(c_j^k, E) < 0 \}$$

Cash flows triggered by current defaults

The cascade ends after at most $n - 1$ rounds: $D_{n-1}(A)$ is the set of defaults generated by the initial default of $A$. 
DEFAULT IMPACT of a financial institution

Case of a single default: $D_{n-1}(i) =$ cascade generated by default of $i$.

We define the “default impact” $DI(i, c, l, E)$ of $i$ as the total loss (in $\) along the default cascade initiated by $i$.

$$DI(i, c, l, E) = \sum_{j \in D_{n-1}(i)} (c_j + \sum_{v \notin D_{n-1}(i)} E_{jv})$$

$DI(i, c, l, E)$ depends -in a deterministic way- on the network structure: matrix of exposures $[E_{jk}]$, liquidity reserves $l_j$, capital $c_j$.

$DI(i, c, l, E)$ is a worst-case loss estimate and does not involve estimating the default probability of $i$. 

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A measure of systemic importance: the Contagion index
(Cont, Moussa, Santos 2010)

- Idea: measure the joint effect of economic shocks and contagion by measuring the Default Impact of a node in a macroeconomic stress scenario
- Apply a common shock $Z$ (in % capital loss) to all balance sheets, where $Z$ is some positive random variable
- Compute Default Impact of node $k$ in this scenario:
  $$\text{DI}(k, c(1-Z), E)$$
- Average across stress scenarios:
  $$\text{CI}(k) = E[ \text{DI}(k, c(1-Z), E) ]$$

Forward-looking, based on exposures and stress scenarios
Contagion and systemic impact of a group of institutions

Similarly we can define the contagion index of a set $A \subset V$ of financial institutions: it is the expected loss to the financial systems generated by the joint default of all institutions in $A$:

$$S(A) = E[DI(A, c + \epsilon, E) | \forall i \in A, c_i + \epsilon_i \leq 0]$$

$S$ then defines a set function

$$S : \mathcal{P}(V) \mapsto \mathbb{R}$$

which associates to each group $A$ of institutions a number quantifying the loss inflicted to the financial system if institutions in the set $A$ fail.

The Systemic Risk Index can be viewed, from the point of view of the regulator, as a macro-level “risk measure”.

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Figure 9: Heterogeneity of systemic importance: distribution of default impact and contagion index across institutions, Brazilian network, 2007-2008.
Figure 10: Evidence for contagion: *contagion index* can be more than thirteen times the *default impact* for some nodes.
Comparison of various capital requirement policies: (a) imposing a minimum capital ratio for all institutions in the network, (b) imposing a minimum capital ratio only for the 5% most systemic institutions, (c) imposing a capital-to-exposure ratio for the 5% most systemic institutions.
The relevance of asymptotics

Most financial networks are characterized by a large number of nodes: FDIC data include several thousands of financial institutions in the US.

To investigate contagion in such large networks, in particular the scaling of contagion effects with size, we can embed a given network in a sequence of networks with increasing size and studying the behavior/scaling of relevant quantities (cascade size, total loss, impact of regulatory policy) when network size increases.

A probabilistic approach consists in
- building an ensemble of random networks of which our network can be considered a typical sample
- showing a limit result (convergence in probability or almost sure convergence) of the relevant quantities in the ensemble considered as $n \to \infty$
Analysis of cascades in large networks

We describe the topology of a large network by the joint distribution $\mu_n(j, k)$ of in/out degrees and assume that $\mu_n$ has a limit $\mu$ when graph size increases in the following sense:

1. $\mu_n(j, k) \to \mu(j, k)$ as $n \to \infty$: the proportion of vertices of in-degree $j$ and out-degree $k$ tends to $\mu(j, k)$.

2. $\sum_{j,k} j \mu(j, k) = \sum_{j,k} k \mu(j, k) =: m \in (0, \infty)$ (finite expectation property);

3. $m(n)/n \to m$ as $n \to \infty$ (averaging property).

4. $\sum_{i=1}^{n} (d_{n,i}^+)^2 + (d_{n,i}^-)^2 = O(n)$ (second moment property).
Contagion in large counterparty networks: analytical results

• Amini, Cont, Minca (2011): mathematical analysis of the onset and magnitude of contagion in a large counterparty network.

• By analogy with models of epidemic contagion, we show that default contagion may become large-scale if

\[
\sum_{j,k} \frac{\mu(j,k) jk}{\lambda} \cdot p(j,k,1) > 1
\]

where

\( \mu(j,k) \) = proportion of nodes with \( j \) debtors, \( k \) creditors

\( \lambda \) = average number of counterparties

\( p(j,k,1) \) : fraction of overexposed nodes with \( (j,k) \) links
Resilience to contagion This leads to a condition on the network which guarantees absence of contagion:

**Proposition 2** (Resilience to contagion). Denote \( p(j, k, 1) \) the proportion of contagious links ending in nodes with degree \((j, k)\). If

\[
\sum_{j,k} k \frac{\mu(j,k)}{\lambda} j p(j, k, 1) < 1
\]

then with probability \( \rightarrow 1 \) as \( n \rightarrow \infty \), the default of a finite set of nodes cannot trigger the default of a positive fraction of the financial network.
Resilience condition:

\[
\sum_{j,k} k \frac{\mu(j, k)}{\lambda} p(j, k, 1) < 1
\] (12)

This leads to a decentralized recipe for monitoring/regulating systemic risk: monitoring the capital adequacy of each institution with regard to its largest exposures.

This result also suggests that one need not monitor/know the entire network of counterparty exposures but simply the skeleton/subgraph of contagious links.

It also suggests that the regulator can efficiently contain contagion by focusing on fragile nodes -especially those with high connectivity- and their counterparties (e.g. by imposing higher capital requirements on them to reduce \( p(j, k, 1) \)).
Simulation-free stress testing of banking systems

- These theoretical results may be used to devise a stress test for the resilience to contagion of a banking system, without the need for large scale simulation.
- We apply a common macro-shock $Z$, measured in % loss in asset value, to all balance sheets in network
- The fraction $p(j,k,Z)$ of *overexposed* nodes with $(j,k)$ links is then an increasing function of $Z$
- Network remains resilient as long as $\sum_{j,k} \frac{\mu(j,k)jk}{\lambda} p(j,k,Z) > 1$
Implications for data collection

- Network analysis points to the importance (for regulators) of observing \textit{counterparty exposures}.

- The example of Brazil shows the feasibility of collecting such data.

- In many countries, exposures larger than a threshold are required to be reported. Our analysis suggest that the relevant threshold should be based on a ratio of the exposure to the capital, not.

- Derivatives exposures (in particular credit derivatives positions) should be reported in more detail (not aggregated mark-to-market value) - notional, underlying, maturity,..- to enable liquidity stress tests of scenarios where margin calls or derivative payoffs re triggered.

- In all countries banks and various financial institutions are
required already to report risk measures (VaR, etc.) on a periodical basis to the regulators.

- Our approach would require these risk figures to be a *disaggregated* across large counterparties: banks would report a figure for exposures to each large counterparty.

- Large financial institutions *already* compute such exposures on a regular basis so requiring them to be reported is not likely to cause a major technological obstacle.

- In principle *any* counterparty is relevant, not just banks.

- On the other hand, only the largest exposures of an institution come into play.
Some conclusions

- Network models provide useful framework for analyzing contagion of default via insolvency/illiquidity across interlinked financial institutions.
- Financial networks are highly heterogeneous across links (exposures) and nodes (connectivity, size): simple, homogeneous/unweighted networks may provide wrong insights on systemic risk and its control.
- Pay attention to the risk measures, not just the model: due to strong heterogeneity, assessments of contagion risk based on cross-sectional averages do not reflect the contribution of contagion to systemic risk.
- Asymptotic analysis of large networks allows to derive explicit mathematical results about the relation between network structure and resilience to contagion for arbitrary networks, which explain results of large-scale simulations. -> (Hamed AMINI’s talk tomorrow afternoon)
- Heterogeneity entails that targeted capital requirements –focusing on the most systemic institutions - are more effective for reducing systemic risk.
- Disaggregating capital ratios: Results suggest that monitoring/putting limits on size and concentration of exposures (as % of capital) is more important than capital ratios based on aggregate asset size.
Some references